

The coincidence problem in the scenario of dark energy interacting with two fluids

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A cosmological model of dark energy interacting with dark matter and another general component of the universe is considered. The evolution equations for coincidence parameters r and s , which represent the ratios between the dark energy and the matter and the other cosmic fluid, respectively, are analyzed in terms of the stability of stationary solutions. The obtained general results allow to shed some light on the coincidence problem and in the equations of state of the three interacting fluids, due to the constraints imposed by the stability of the solutions. We found that for an interaction proportional to the sum of the DE density and the third fluid density, the hypothetical fluid must have positive pressure, which leads naturally to a cosmological scenario with radiation, unparticle or even some form of warm DM as the third interacting fluid.

I. INTRODUCTION

The existence of a dark component with an exotic equation of state, i. e., with a ratio $w = p/\rho$ negative and close to -1 , which drives an accelerated expansion is consistent with the luminosity distance as a function of redshift of distant supernovae [1], the structure formation (LSS) [2] and the cosmic microwave background (CMB) [3]

The cosmic observations show that densities of dark energy and dark matter are of the same order today. To solve this coincidence problem [4] (or why we are accelerating in the current epoch due that the vacuum and dust energy density are of the same order today ?) it is assumed an evolving dark energy field with a non-gravitational interaction with matter [5] (decay of dark energy to matter).

Although the main topic of investigation have been centered in the interactions in the dark sector, it is physically reasonable and even expected from a theoretical point of view, that dark components can interact with other fluids of the universe. For example DE interacting with neutrinos was investigated in [6], and decaying into the fermion fields, in [7]. A more general scenario was considered in [8], in which DE is interacting with neutrinos and DM.

Inspired in these previous investigation we have recently formulated a effective model where DE is decaying in DM and another hypothetical fluid [9], which we shall denote here and after by DX. In the framework of the holographic DE, and using the Hubble radius as infrared cutoff, we have shown that our scenario leads naturally, for a flat universe, to a more suitable approach to the cosmic coincidence problem in which the ratio between the energy densities of DM and DE, r , can be variable during the cosmic evolution. Our model has been discussed has a possible approach to solve the triple coincidence problem in [10]. In this work it was assumed that the third fluid is radiation. Although matter and radiation are almost non-interacting fluids, since the decoupling era, nevertheless they could interact with DE. In [11] was investigated the dynamical behavior when DE is coupling to DM and unparticle in the flat FRW cosmology.

The aim of this paper is to investigate further the model of DE interacting with DM and another hypothetical fluid. Despite of the interesting results found in [10, 11] when this third

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fluid was specifically identified with radiation and unparticle, we continue assuming only that this fluid has an equation of state with ω constant. We expect, studying the stationary solutions of the evolution equation for r and s , the ratio between the energy densities of DE and DX, to found suitable constraints for the equation of state of the unknown fluid. Since we are introducing a third fluid in the interactions in the dark sector, we generalize the coupling terms that have been already considered in the literature, although an interaction term which include the energy density of unparticle was discussed in [11]. In this paper we choose three different coupling terms in order to investigate the dynamical behavior of the models of DE interacting with DM and a third unknown fluid.

Our paper is organized as follows. In section II we present the model for a universe filled with dark matter, dark energy and another fluid. We shall impose that the interacting term δ and δ' which appears in the conservation equations are different. In section III we study the stationary solutions and its stability for the equations of evolution of the ratios between the DM and DE and the third fluid and DE. We presents the constraints on the equation of state for DE and the hypothetical third fluid. In section IV we discuss our results.

II. INTERACTING DARK ENERGY

In the following we modeled the universe made of CDM with a density ρ_m and a dark energy component, ρ_D . We will assume that the the dark matter component is interacting with the dark energy component, so their continuity equations take the form

$$\dot{\rho}_D + 3H(1 + \omega_D)\rho_D = -\delta', \quad (1)$$

$$\dot{\rho}_m + 3H(1 + \omega_m)\rho_m = \delta. \quad (2)$$

where δ and δ' are the interactions terms, which are different in order to include the scenario in which the mutual interaction between the two principal components of the universe leads to some loss in other forms of cosmic constituents. We assume that the interactions terms are of the form $\delta = 3H\Gamma$, where $\Gamma = \Gamma(\rho_D, \rho_m, \rho_X)$. Since we will study an universe with dark energy decaying into dark matter, we have from the beginning $\delta > 0$ and $\delta' > 0$. If we denote this other component by ρ_X , its corresponding continuity equation is given by

$$\dot{\rho}_X + 3H(1 + \omega_X)\rho_X = \delta' - \delta = 3H\Pi, \quad (3)$$

where $\Pi = \Gamma' - \Gamma$. We are taking about in this case that dark energy decay into dark matter (or viceversa, depending on the sign of δ) and other component. The sourced Friedmann equation is then given by

$$3H^2 = \rho_D + \rho_m + \rho_X - \frac{3k}{a^2}. \quad (4)$$

Since we are interested in the three cosmic coincidence, we do not we assume as it was done in [9], that $\rho_D \gg \rho_X$ and $\rho_m \gg \rho_X$. In order to study the evolution of the densities of these three fluids we construct the differential equations for the coincidence parameters $r = \rho_m/\rho_D$ and $s = \rho_X/\rho_D$. These equations takes the following expressions

$$r' = \frac{\dot{r}}{H} = \frac{r}{H} \left(\frac{\dot{\rho}_m}{\rho_m} - \frac{\dot{\rho}_D}{\rho_D} \right), \quad (5)$$

and

$$s' = \frac{\dot{s}}{H} = \frac{s}{H} \left(\frac{\dot{\rho}_X}{\rho_X} - \frac{\dot{\rho}_D}{\rho_D} \right). \quad (6)$$

We assume that the interaction term takes the general form

$$\Gamma = \lambda_1 \rho_D + \lambda_2 \rho_m + \lambda_3 \rho_X = (\lambda_1 + \lambda_2 r + \lambda_3 s) \rho_D. \quad (7)$$

Consistently, for the general form given in Eq.(7), we can parameterized $\Pi = \Gamma' - \Gamma$ through-out suitable constants $\lambda_1^\pi, \lambda_2^\pi, \lambda_3^\pi$, in the following form

$$\Pi = \lambda_1^\pi \rho_D + \lambda_2^\pi \rho_m + \lambda_3^\pi \rho_X = (\lambda_1^\pi + \lambda_2^\pi r + \lambda_3^\pi s) \rho_D. \quad (8)$$

Notice that the form of interaction considered here is a generalization of the cases previously investigated, since the interactions taken account consider only functions of the dark sector densities. Interactions that are linear combinations of the dark sector densities have been studied in [12], when the interaction is only between the dark components.

Using Eqs.(1) and (2) in the equations (5) and (6), with the interaction terms, Γ and Π given by Eqs. (7) and (8), respectively, we obtain the evolution equations for the parameters r and s

$$r' = 3r [(\lambda_1 + \lambda_2 r + \lambda_3 s)(1 + 1/r) + (\lambda_1^\pi + \lambda_2^\pi r + \lambda_3^\pi s) + \omega_D - \omega_m], \quad (9)$$

and

$$s' = 3s [\lambda_1 + \lambda_2 r + \lambda_3 s + (1 + 1/s)(\lambda_1^\pi + \lambda_2^\pi r + \lambda_3^\pi s) + \omega_D - \omega_X]. \quad (10)$$

III. STATIONARY SOLUTIONS

Before to look for the stationary solutions of Eqs.(5) and (6), let us briefly discuss the what we assume for the equations of state of the three cosmic fluids that are under interaction. At this stage, we only consider that one fluid, with an energy density ρ_D and equation of state ω_D , is decaying into the other two fluids. Although, it is reasonable to take $\omega_m = 0$ from the beginning, since we are thinking in the dark matter fluid, we shall postpone this election until to obtain the constraint derived from the study of the stationary solutions of Eqs.(5) and (6) and its stability. For simplicity, the three equations of state are taken constant.

Since we are looking for stationary solutions of Eqs.(5) and (6) we set $r' = s' = 0$, obtaining a systems of algebraic equations in terms of the the variables r and s , with the parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_1^\pi, \lambda_2^\pi, \lambda_3^\pi, \omega_D, \omega_X$. In order to study how suitable interactions leads to these three cosmic fluids to have stationary solutions we begin with the simplest cases.

A. Case $\Gamma = \lambda\rho_D$ and $\Pi = \lambda^\pi\rho_D$

In this case we are choosing $\lambda_2 = \lambda_3 = \lambda_2^\pi = \lambda_3^\pi = 0$. The interactions terms are proportional to the dark energy density. This type of interaction has been investigated in [11, 13, 14]. The condition $r' = s' = 0$ leads to the algebraic systems

$$f(r, s)|_{r=r_s, s=s_s} = \lambda(1 + r_s) + \lambda^\pi r_s + (\omega_D - \omega_m)r_s = 0. \quad (11)$$

$$g(r, s)|_{r=r_s, s=s_s} = \lambda s_s + \lambda^\pi(1 + s_s) + (\omega_D - \omega_X)s_s = 0. \quad (12)$$

Notice that the interaction assumed gives two simple linear equation non acopled in the variables r_s and s_s , which are the stationary solutions given by

$$r_s = -\frac{\lambda}{\lambda + \lambda^\pi + \omega_D - \omega_m}, \quad (13)$$

and

$$s_s = -\frac{\lambda^\pi}{\lambda + \lambda^\pi + \omega_D - \omega_X}. \quad (14)$$

Since r_s and s_s are positive quantities, the denominators of the above both equations must be negative, so be obtain the following inequalities

$$\omega_D < \omega_m - (\lambda + \lambda^\pi), \quad (15)$$

and

$$\omega_X > \omega_D + (\lambda + \lambda^\pi). \quad (16)$$

In order to study the stability of the found solutions we shall evaluate the eigenvalues of the matrix \mathbf{M}

$$\mathbf{M} = \begin{pmatrix} \frac{\partial f(r,s)}{\partial r} & \frac{\partial f(r,s)}{\partial s} \\ \frac{\partial g(r,s)}{\partial r} & \frac{\partial g(r,s)}{\partial s} \end{pmatrix} \quad (17)$$

whose elements are evaluate at the critical point (r_s, s_s) . From the equation for the eigenvalues, $\det[\mathbf{M} - \eta \mathbf{I}]$, and since $\partial f(r, s)/\partial s = \partial g(r, s)/\partial r = 0$, we obtain

$$\left[\left(\frac{\partial f(r, s)}{\partial r} \right)_{|r=r_s, s=s_s} - \eta \right] \left[\left(\frac{\partial g(r, s)}{\partial s} \right)_{|r=r_s, s=s_s} - \eta \right] = 0. \quad (18)$$

The eigenvalues are then

$$\eta_1 = \left(\frac{\partial f(r, s)}{\partial r} \right)_{|r=r_s, s=s_s} = \lambda + \lambda^\pi + \omega_D - \omega_m \quad (19)$$

and

$$\eta_2 = \left(\frac{\partial g(r, s)}{\partial s} \right)_{|r=r_s, s=s_s} = \lambda + \lambda^\pi + \omega_D - \omega_X \quad (20)$$

The condition of stability, $\eta_1 < 0$ and $\eta_2 < 0$ is the same just contained in Eqs.(15) and (16). So, if for a given ω_D , ω_m , ω_X , λ , λ^π there are positive stationary solutions, they are also stable. For the case of dark matter with negligible pressure, i.e., $\omega_m = 0$, Eq.(15) implies that the dark energy must necessarily have an equation of state with $\omega_D < 0$. A rough estimation of the value of the sum $\lambda + \lambda^\pi$ can be obtained from the equation for the acceleration

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(1 + 3\omega)\rho, \quad (21)$$

where $\rho \equiv \rho_D(1 + r + s)$ and $\omega \equiv (\omega_D + \omega_m r + \omega_X s)/(1 + r + s)$. If an accelerated phase is demanded we obtain the following inequality

$$\omega_D < -\frac{1}{3} [1 + (1 + 3\omega_m)r + (1 + 3\omega_X)s]. \quad (22)$$

In terms of the parameters Ω_D and Ω_X (for $\omega_m = 0$) we obtain

$$\omega_D < -\frac{1}{3\Omega_D}(1 + 3\omega_X\Omega_X). \quad (23)$$

If it is assumed that $\Omega_X \ll 1$, as we expect for today for any other fluid different from the dark sector. In order to obtain a rough estimation of the term $\lambda + \lambda^\pi$, we equate the right hand side of the expressions (15) and (23), and taking $\Omega_D = 0.7$ we obtain

$$\lambda + \lambda^\pi \simeq 0.5, \quad (24)$$

so for $\omega_D \lesssim -0.5$ the three interacting fluids leads to stationary and stable solutions for r and s . From the inequalities (15) and (16) we obtain that $\omega_X \leq \omega_m$. So for $\omega_m = 0$ the third fluid could be a normal fluid or even an exotic fluid. Nevertheless, from the expression for the ratio $\frac{s_s}{r_s}$, given by

$$\frac{s_s}{r_s} = \frac{\lambda^\pi}{\lambda} \left(1 - \frac{\omega_X}{\omega_D + \lambda + \lambda^\pi} \right)^{-1}, \quad (25)$$

we can conclude, assuming $\frac{s_s}{r_s} \ll 1$, that an scenario with $\omega_X > 0$ is more suitable taking $\lambda \gg \lambda^\pi$. From Eq. (13) and taking $r_s \approx 0.3/0.7$, $\omega_D \simeq -1$, we obtains for the realistic case $\lambda \gg \lambda^\pi$ that $\lambda \simeq 0.3$.

B. Case $\Gamma = \lambda\rho_m$ and $\Pi = \lambda^\pi\rho_m$

In this case we are choosing $\lambda_1 = \lambda_3 = \lambda_1^\pi = \lambda_3^\pi = 0$. The interactions terms are proportional only to the dark matter density. This type of interaction was investigated for models of interacting phantom dark energy with dark matter [11, 15, 16, 17] and also in [14, 18, 19]. Observational constraints on λ for this type of interaction have been investigated in [20]. The condition $r' = s' = 0$ leads to the algebraic systems

$$f(r, s)|_{r=r_s, s=s_s} = r_s^2(\lambda + \lambda^\pi) + r_s(\lambda + \omega_D - \omega_m) = 0. \quad (26)$$

$$g(r, s)|_{r=r_s, s=s_s} = s_s r_s(\lambda + \lambda^\pi) + \lambda^\pi r_s + (\omega_D - \omega_X)s_s = 0. \quad (27)$$

In this case the interaction assumed gives two coupled non linear equations in the variables r_s and s_s . The Eq. (26) has the following solution different from zero ,

$$r_s = -\frac{\lambda + \omega_D - \omega_m}{\lambda + \lambda^\pi}. \quad (28)$$

Imposing the condition $r_s > 0$ we obtain the constraint

$$\lambda + \omega_D - \omega_m < 0. \quad (29)$$

Introducing the value for r_s , given by Eq.(28), in the Eq.(27) yields

$$s_s = \frac{\lambda^\pi}{\lambda + \omega_m - \omega_X} \left(\frac{\lambda + \omega_D - \omega_m}{\lambda + \lambda^\pi} \right). \quad (30)$$

Using the constrain given in Eq.(29) in the expression for s_s we obtain that $s_s > 0$ implies

$$-\lambda + \omega_m - \omega_X < 0. \quad (31)$$

From the equation for the eigenvalues, $\det[\mathbf{M} - \eta \mathbf{I}]$, and since $\partial f(r, s)/\partial s = 0$, we obtain

$$\left[\left(\frac{\partial f(r, s)}{\partial r} \right) \Big|_{r=r_s, s=s_s} - \eta \right] \left[\left(\frac{\partial g(r, s)}{\partial s} \right) \Big|_{r=r_s, s=s_s} - \eta \right] = 0. \quad (32)$$

The eigenvalues are then

$$\eta_1 = \left(\frac{\partial f(r, s)}{\partial r} \right) \Big|_{r=r_s, s=s_s} = 2r_s(\lambda + \lambda^\pi)(\lambda + \omega_D - \omega_m) \quad (33)$$

and

$$\eta_2 = \left(\frac{\partial g(r, s)}{\partial s} \right) \Big|_{r=r_s, s=s_s} = r_s(\lambda + \lambda^\pi) + \omega_D - \omega_X \quad (34)$$

Notice that the condition of stability, $\eta_1 < 0$ and $\eta_2 < 0$ gives us, for $\eta_1 < 0$ the following constraint

$$\lambda + \omega_D - \omega_m > 0, \quad (35)$$

which can not be allowed if Eq.(29) is satisfied. Then for this type of interaction it is not possible to have stable solutions for three cosmic interacting fluids. Nevertheless, this situation is a consequence of chose $\lambda > 0$ and $\lambda^\pi > 0$ from the beginning. It is straightforward to prove that for $\lambda + \lambda^\pi < 0$ and $\lambda^\pi > 0$, or equivalently $\lambda < 0$ and $|\lambda| > |\lambda^\pi|$, the fixed point of the system are stable. The constraints for the equations of state are the following

$$\lambda + \omega_D - \omega_m > 0, \quad (36)$$

and

$$-\lambda + \omega_D - \omega_X < 0, \quad (37)$$

which for $\omega_m = 0$ yields $\lambda + \omega_D > 0$ and $\lambda + \omega_X > 0$. Since $\lambda < 0$ the above conditions implies a third fluid with $\omega_X > 0$ and also a DE with $\omega_D > 0$. So in the approach of DE interacting with two fluids, this kind of coupling give stable solutions but a cosmic evolution without acceleration.

C. Case $\Gamma = \lambda\rho_D(1+s)$ and $\Pi = \lambda^\pi\rho_D(1+s)$

In this case we have taken $\lambda_1 = \lambda_3$, $\lambda_2 = \lambda_2^\pi = 0$, and $\lambda_1^\pi = \lambda_3^\pi$. The interaction is, in this case, proportional to $\rho_D + \rho_X$. A coupling term which include a different fluid from those of the dark sector was already introduced in [11], but throughout expressions like $\rho_D\rho_X$ and $\rho_m\rho_X$, where DX was identified with unparticle. The condition $r' = s' = 0$ leads to the following algebraic system

$$f(r, s)|_{r=r_s, s=s_s} = \lambda(1+r_s)(1+s_s) + \lambda^\pi r_s(1+s_s) + (\omega_D - \omega_m)r_s = 0. \quad (38)$$

$$g(r, s)|_{r=r_s, s=s_s} = \lambda s_s(1+s_s) + \lambda^\pi(1+s_s)^2 + (\omega_D - \omega_X)s_s = 0. \quad (39)$$

Solving first the Eq.(39), which is a second grade equation of the form $x^2 + Bx + C = 0$, where the coefficients B and C are given by

$$B = \frac{\lambda + 2\lambda^\pi + \omega_D - \omega_X}{\lambda + \lambda^\pi}; \quad C = \frac{\lambda^\pi}{\lambda + \lambda^\pi}. \quad (40)$$

The solutions of Eq.(39) has the form

$$s_s = \frac{B}{2} \left(-1 \pm \sqrt{1 - \frac{4C}{B^2}} \right). \quad (41)$$

Since s_s is a positive and real number we need to impose the two constraints $B < 0$ and $B^2 > 4C$. The first one implies that $\lambda + 2\lambda^\pi + \omega_D - \omega_X < 0$ and the second one, $\lambda + 2\lambda^\pi + \omega_D - \omega_X > -2\sqrt{\lambda^\pi(\lambda + \lambda^\pi)}$, which gives the following range for $\omega_D - \omega_X$

$$- \left((\lambda + 2\lambda^\pi) + 2\sqrt{\lambda^\pi(\lambda + \lambda^\pi)} \right) < \omega_D - \omega_X < -(\lambda + 2\lambda^\pi). \quad (42)$$

Introducing the two solutions of Eq.(39), which we denote by s_{s+} and s_{s-} , in Eq.(38), we obtain two solutions for r_s , r_{s+} and r_{s-} , given by

$$r_{s+} = -\frac{\lambda}{\lambda + \lambda^\pi + \frac{\omega_D - \omega_m}{1+s_{s+}}}; \quad r_{s-} = -\frac{\lambda}{\lambda + \lambda^\pi + \frac{\omega_D - \omega_m}{1+s_{s-}}} \quad (43)$$

Since $r_s > 0$ Eq.(43) gives the following constraint

$$(\lambda + \lambda^\pi)(1 + s_{s\pm}) + \omega_D - \omega_m < 0, \quad (44)$$

so ω_D must satisfy

$$\omega_D < \omega_m - (\lambda + \lambda^\pi)(1 + s_{s\pm}). \quad (45)$$

The conditions which are necessary to hold in order to have positive solutions for $r_{s\pm}$ and $s_{s\pm}$ are then the inequalities given by Eqs. (42) and (44)

As in the case **A**, where the interaction is proportional only to the DE density, if $\omega_m = 0$ the above inequality implies that the dark energy must necessarily have an equation of state with $\omega_D < 0$. A rough estimation for the range of the values that the parameters λ and λ^π can take, may also be done for this case using Eq. (23) with $\Omega_X \ll 1$ and $\Omega_D = 0.7$. Equating the right hand side of the expressions (45) and (23) and since $1 + s_{s\pm} > 1$

$$\lambda + \lambda^\pi < 0.5. \quad (46)$$

Let us to study now the conditions imposed on the found solutions if we require stability. Evaluating the elements of the matrix **M** at the critical points (r_{s+}, s_{s+}) and (r_{s-}, s_{s-}) , we obtain from the equation for the eigenvalues, $\det[\mathbf{M} - \eta \mathbf{I}]$, and since $\partial g(r, s)/\partial r = 0$, that

$$\left[\left(\frac{\partial f(r, s)}{\partial r} \right) \Big|_{r=r_{s\pm}, s=s_{s\pm}} - \eta \right] \left[\left(\frac{\partial g(r, s)}{\partial s} \right) \Big|_{r=r_{s\pm}, s=s_{s\pm}} - \eta \right] = 0. \quad (47)$$

The eigenvalues are then

$$\eta_1 = \left(\frac{\partial f(r, s)}{\partial r} \right) \Big|_{r=r_{s\pm}, s=s_{s\pm}} = (\lambda + \lambda^\pi)(1 + s_{s\pm}) + \omega_D - \omega_m, \quad (48)$$

and

$$\eta_2 = \left(\frac{\partial g(r, s)}{\partial s} \right) \Big|_{r=r_{s\pm}, s=s_{s\pm}} = (\lambda + 2\lambda^\pi)(1 + s_{s\pm}) + \lambda s_{s\pm} + \omega_D - \omega_X. \quad (49)$$

The condition of stability, $\eta_1 < 0$ and $\eta_2 < 0$, implies that the following constraints must hold

$$\omega_D - \omega_m < -(\lambda + \lambda^\pi)(1 + s_{s\pm}), \quad (50)$$

and

$$\omega_D - \omega_X < -[(\lambda + 2\lambda^\pi)(1 + s_{s\pm}) + \lambda s_{s\pm}]. \quad (51)$$

Notice that the constraint given by Eq.(50) is the same obtained in Eq.(44). Nevertheless, we need to look for the range of $\omega_D - \omega_X$ which can accommodate the constraint given by Eq.(42) and Eq.(51). Choosing Eq.(51) as the constraint for the upper limit of $\omega_D - \omega_X$, the upper limit indicated in Eq.(42) is also satisfied. We can impose the condition

$$- \left((\lambda + 2\lambda^\pi) + 2\sqrt{\lambda^\pi(\lambda + \lambda^\pi)} \right) < -[(\lambda + 2\lambda^\pi)(1 + s_{s\pm}) + \lambda s_{s\pm}], \quad (52)$$

which leads to the following condition for $s_{s\pm}$

$$s_{s\pm} < \sqrt{\frac{\lambda^\pi}{\lambda + \lambda^\pi}}. \quad (53)$$

Since it is physically reasonable assume $s_{s\pm} \ll 1$ for a late time evolution,

It is straightforward to check from the expressions Eq.(50) and Eq.(51) that, independently of the critical point considered, the equation of state of the non decaying fluids satisfy

$$\omega_X > \omega_m. \quad (54)$$

If $\omega_m = 0$, which is the equation of state for the dark matter fluid, the above result indicates us that the exigency of stability for the stationary solutions of the evolution equations (9) and (10), imposes an unknown interacting fluid with non null pressure.

IV. DISCUSSION

In the present investigation we have considered a cosmological scenario where the dark energy is decaying into the dark matter and another component of the universe, which we do not identify explicitly. We have assumed that this three fluids have an equation of state with ω constant. We have choose three different coupling terms, analyzing the stationary solutions of the evolution equation for parameters r and s .

When the coupling is proportional to the dark energy only, we have found that the conditions for the stationary solutions be positive are the same of those to be stable. For dark matter with negligible pressure we obtain that the dark energy must necessarily have an equation of state with $\omega_D < 0$.

When the coupling is only proportional to the DM energy density, since in this case it is not possible to obtain stable solutions for the three interacting fluids, if $\lambda > 0$ and $\lambda^\pi > 0$, which guaranty that DE decaying in the other fluids. Nevertheless, relaxing this condition and taking $\lambda + \lambda^\pi < 0$ and $\lambda^\pi > 0$ we obtain stationary solutions which are stable. Notice that this physically correspond to a DM decaying in DE and in the third fluid. It is interesting to mention that the usual case of DE interacting only with DM, which have been discussed for this coupling in [20], showed that the data slightly favored a DM decaying in DE and $\omega_D < -1$. Unfortunately in our approach, we obtain that the unknown fluid and DE have positive pressure, leading to a decelerated expansion. As in the case of DE interacting only with DM, there is no stable solutions for DE with negative pressure.

In the third case studied, the coupling consider a different fluid from those of the dark sector, taking a term proportional to the sum of the DE density and the third fluid density. We have found two fixed points and from the constraints derived from the condition of stability we have obtain that $\omega_X > \omega_m$, which means an interacting third fluid with positive pressure. This type of coupling can then accommodate an scenario with radiation [10], unparticle [11] or even some form of warm DM [21] as the third interacting fluid.

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- [1] A. G. Riess et al. [Supernova Search Team Collaboration], *Astron. J.* 116 (1998) 1009; S. Perlmutter et al. [Supernova Cosmology Project Collaboration], *Astrophys. J.* 517 (1999) 565; W. J. Percival et al. [The 2dFGRS Collaboration], *Mon. Not. Roy. Astron. Soc.* 327 (2001) 1297; P. Astier et al., [astro-ph/0510447]; A.G.Riess et al. [Supernova Search Team Collaboration], *Astrophys. J.* 607, 665 (2004).
 - [2] W. J. Percival et al. [The 2dFGRS Collaboration], *Mon. Not. Roy. Astron. Soc.* 327, 1297 (2001); M. Tegmark et al. [SDSS Collaboration], *Phys. Rev. D* 69 (2004) 103501.
 - [3] D. N. Spergel et al., [astro-ph/0603449]; A. C. S. Readhead et al., *Astrophys. J.* 609 (2004) 498; J. H. Goldstein et al., *Astrophys. J.* 599, 773 (2003) ; E. Komatsu et al, [astro-ph/0803.0547].
 - [4] S. Weinberg, *Rev. Mod. Phys.* 61, 1 (1989); I. Zlatev, L. M. Wang and P. J. Steinhardt, *Phys. Rev. Lett.* 82, 896 (1999).
 - [5] L. Amendola, *Phys. Rev. D* 62, 043511 (2000); W. Zimdahl, D. Pavón and L.P. Chimento, *Phys. Lett. B* 521, 133 (2001); M. Gasperini, F. Piazza and G. Veneziano, *Phys. Rev. D* 65, 023508 (2002); W. Zimdahl, *Int. J. Mod. Phys. D* 14, 2319 (2005); D. Pavón and W. Zimdahl, *Phys. Lett. B* 628, 206 (2005); G. Mangano, G. Miele and V. Pettorino, *Mod. Phys. Lett. A* 18,

- 831 2003); G. Farrar and P.J.E. Peebles, *Astrophys. J.* 604, 1 (2004); S. del Campo, R. Herrera and D. Pavón, *Phys. Rev. D* 70, 043540 (2004); R. Cai and A. Wang, *J. Cosmol. Astropart. Phys.* 03, 002 (2005); Micheal S. Berger and H. Shojaei, *Phys. Rev. D* 73, 083528 (2006); Bo Hu and Y. Ling, *Phys. Rev. D* 73, 123510 (2006); Hui Li, Z. Guo and Y. Zhang, *Int. J. Mod. Phys. D* 15, 869 (2006); A. P. Billyard and A. A. Coley, *Phys. Rev. D* 61, 083503 (2000); M. Szydlowski, *Phys. Lett. B* 632, 1 (2006); M. Szydlowski, T. Stachowiak and R. Wojtak, *Phys. Rev. D* 73, 063516 (2006); L. P. Chimento, A. S. Jakubi, D. Pavón and W. Zimdahl, *Phys. Rev. D* 67, 083513 (2003); L. P. Chimento and D. Pavón, *Phys. Rev. D* 73, 063511 (2006); G. Olivares, F. Atrio-Barandela and D. Pavón, [arXiv:gr-qc/0601086]; L. Amendola, G. Camargo Campos and R. Rosenfeld [arXiv:gr-qc/061006].
- [6] A. W. Brokfield, C. van de Bruck, D. F. Mota and D. Tocchini-Valentini, *Phys. Rev. Lett.* 96, 061301 (2006); *Phys. Rev. D* 73 085515 (2006).
- [7] A. de la Macorra, [arXiv:astro-ph/0702239]; [arXiv:astro-ph/0701635].
- [8] G. M. Kremer, [arXiv:gr-qc/0704.0371].
- [9] N. Cruz, S. Lepe and F. Peña, *Phys. Lett. B* 603, 1 (2008).
- [10] M. Jamil and F. Rahaman, [arXiv:gr-qc/0810.1444].
- [11] S. Chen and J. Jing, [arXiv:gr-qc/0903.0120].
- [12] M. Quartin, M. O. Calvao, S. E. Joras, R.R.R. Reis and I. Waga, *JCAP* 0805, 007 (2008); G. Caldera-Cabral, R. Maartens and L. Arturo Ureña-López, [arXiv:gr-qc/0812.1827]; H. M. Sadjadi and M. Alimohammadi, *Phys. Rev. D* 74, 103007 (2006).
- [13] S. del Campo, R. Herrera and D. Pavon, *JCAP* 0901, 020 (2009).
- [14] J. H. He and B. Wang, *J. Cosmology Astropart. Phys.* 06, 010 (2008).
- [15] E. Majerotto, D. Sapone and L. Amendola, [arXiv:astro-ph/0410543].
- [16] R. Cai and A. Wang, *JCAP* 0503, 002 (2005).
- [17] X. Chen, Y. Gong and E. N. Saridakis, [arXiv:gr-qc/0812.1117].
- [18] X. Chen, Y. Gong, [arXiv:gr-qc/0811.1698].
- [19] C. G. Bömer, G. Caldera-Cabral, R. Lazkoz and R. Maartens, *Phys. Rev. D* 78, 023505 (2008).
- [20] Z. Guo, N. Ohta and S. Tsujikawa, *Phys. Rev. D* 76, 023508 (2007).
- [21] A. V. Tikhonov, S. Gottlöber, G. Yepes and Y. Hoffman, [arXiv:astro-ph.CO/0904.0175]; J. Fabris, I. L. Shapiro and F. Sobreira, [arXiv:astro-ph/0806.1969]; J.R. Bond and A. S. Szalay, *Astrophys. J.* 274, 443 (1986); P. Bode, J. P. Ostriker and N. Turok, *Astrophys. J.* 556, 93

(2001); S. Colombi, S. Dodelson and L. M. Widrow, *Astrophys. J.* 459, 1 (1996); C. M. Muller, *Phys. Rev D* 71, 047302 (2005).